U.G. 5th Semester Examination - 2021

MATHEMATICS

[HONOURS]

Discipline Specific Elective (DSE)

Course Code : MATH-H-DSE-T-01A

(Linear Programming)

Full Marks : 60 Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

The notations and symbols have their usual meanings.

1. Answer any **ten** questions: 2×10=20

i) When a game is said to be fair? Give an example.

ii) Express the following LPP as standard maximization problem:

Minimize $Z = 4x_1 - x_2 + 2x_3$

Subject to
$$4x_1 + x_2 - x_3 \le 7$$
$$2x_1 - 3x_2 + x_3 \le 12$$
$$x_1 + x_2 + x_3 = 8$$
$$4x_1 + 7x_2 - x_3 \ge 16$$
$$x_1, x_2, x_3 \ge 0.$$

iii) Find the basic feasible solutions of the equation $2x_1 + 3x_2 - x_3 = 6$.

[Turn Over]

iv) Show that intersection of two convex sets is also a convex set.

v) Extreme points are finite in number. Justify.

vi) Prove that the set defined by $X = \{x : |x| \le 2\}$ is a convex set.

vii) Find out the extreme points (if any) of the convex set $S = \{(x, y : |x| \le 1, |y| \le 1)\}$.

viii) Is the point (1,10) lie in the convex set of feasible solutions determined by the constraints $2x_1 + 5x_2 \le 40,\ x_1 + x_2 \le 11,\ x_2 \ge 4,\ x_1, x_2 \ge 0$?

ix) State Complementary slackness theorem of Duality theory.

x) What is the relation between the optimal values of primal and dual problems (assume that both exist)?

xi) How can you detect that in a Transportation Problem the solution is optimal? What is the criterion for the existence of multiple optimal solutions?

xii) What is unbalanced Transportation Problem? How can you convert it into a balanced Transportation Problem?

xiii) Define a symmetric game. Why is it called so?

xiv) State Fundamental theorem of LPP.

xv) Write down the dual of the following problem :

Maximize $Z = 2x_1 + 3x_2 + x_3$

subject to $2x_1 + x_2 - 3x_3 \le 7$

$2x_1 - x_2 + x_3 \le 6$

$x_1 + 3x_2 + x_3 \le 8$

$2x_1 + 3x_2 - x_3 \ge 12$

$x_1, x_2, x_3 \ge 0.$

2. Answer any **four** questions: 5×4=20

i) Solve the following L.P.P. by graphical method:

Maximize $Z = 8000x_1 + 7000x_2$

subject to $3x_1 + x_2 \le 66$

$x_1 + x_2 \le 45$

$x_1 \le 20$

$x_2 \le 40$

$x_1, x_2 \ge 0.$

ii) Show that $x_1 = 5$, $x_2 = 0$, $x_3 = -1$ is a basic solution of the following set of equations

$x_1 + 2x_2 + x_3 = 4$

$2x_1 + x_2 + 5x_3 = 3$

Find the other basic solution, if there be any.

iii) Solve the following LPP by simplex method:

Maximize $Z = 3x_1 + 2x_2$

s.t. $x_1 + x_2 \le 4$

$x_1 - x_2 \le 2$

$x_1, x_2 \ge 0.$

iv) Solve the following balanced Transportation problem :

	D_1	D_2	D_3	D_4	a_i
	9	8	5	7	12
	4	6	8	7	14
	5	8	9	5	16
b_j	8	18	13	3	

v) Find out the optimal (maximum) assignment profit from the following cost matrix:

	I	II	III	IV
A	9	6	6	5
B	8	7	5	6
C	8	6	5	7
D	9	9	8	8

vi) State and prove fundamental duality theorem.

3. Answer any **two** questions: 10×2=20

i) a) Solve the following L.P.P. : 6+4=10

Maximize $Z = 4x_1 + 3x_2$

subject to $3x_1 + x_2 \le 15$

$3x_1 + 4x_2 \le 24$

$x_1, x_2 \ge 0.$

b) Show that

$$S = \{(x_1, x_2, x_3) : 2x_1 - x_2 + x_3 \le 4, x_1 + 2x_2 - x_3 \le 1\}$$

is a convex set.

ii) Solve the following L.P.P. by using Big-M method: 10

Maximize $Z = -2x_1 - x_2$

subject to $3x_1 + x_2 = 3$

$4x_1 + 3x_2 \ge 6$

$x_1 + 2x_2 \le 4$

$x_1, x_2 \ge 0.$

iii) Write down the dual of the following problem:

Minimize $Z = -12x_1 + 6x_2 - 4x_3$

subject to $-3x_1 + x_2 + x_3 \ge 3$

$4x_1 + x_2 + x_3 \le 2$

$x_1, x_2, x_3 \ge 0.$

Solving the dual problem, discuss the nature of the primal problem. 10

iv) a) Prove that, in a balanced transportation problem having m origins and n destinations $(m, n \ge 2)$ the exact number of basic solutions is m+n-1.

b) Reduce the following game to 2×2 game and then solve it : 4+6=10

$$\begin{bmatrix} 3 & 2 & 4 & 0 \\ 3 & 4 & 2 & 4 \\ 4 & 2 & 4 & 0 \\ 0 & 4 & 0 & 8 \end{bmatrix}$$

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